Test 1-3

① arc length $= \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$

$r(t) = \langle \cos t, \sin t, \ln(\cos t) \rangle \quad 0 \le t \le \frac{\pi}{3}$

$\int_0^{\frac{\pi}{3}} \sqrt{(-\sin t)^2 + (\cos t)^2 + (\tan t)^2}\, dt$

$= \ln(\sqrt{3} + 2)$

$r(t) = \langle 2\cos t, 2\sin t, t^2 \rangle \quad t = 0$

② Normal plane $= \langle r'(t) \cdot (x - x_0, y - y_0, z - z_0) \rangle$ $\quad r(0) = \langle 2, 0, 0 \rangle$

$\langle -2\sin t, 2\cos t, 2t \rangle \cdot \langle x - 2, y, z \rangle$

$(-2\sin t)x + 4\sin t + 2y\cos t - 2zt = 0$

@ $t = 0$, $y = 0$

③ Unit tangent vector

$T(t) = \frac{r'(t)}{|r(t)|} = \frac{\langle -2\sin t, 2\cos t, 2t \rangle}{\sqrt{4\sin^2 t + 4\cos^2 t + 4t^2}}$

@ $t = 0$: $\langle -\sin t, \cos t, t \rangle = \langle 0, 1, 0 \rangle$

④ Curvature $= \frac{|T'(t)|}{|r'(t)|} = \frac{\left| \frac{\langle -\cos t, -\sin t, 1 \rangle}{\sqrt{4\sin^2 t + 4\cos^2 t + 4t^2}} \right|}{\sqrt{4\sin^2 t + 4\cos^2 t + 4t^2}}$

$= \frac{\sqrt{2}}{2t} = \frac{\sqrt{2}}{2}$

$\sqrt{4(\sin^2 t + \cos^2 t + t^2)}$

$4(1 + t^2)$

$\sqrt{4t^2 + 4}$

$x^2yz = \sin(x + y - z)$

$f(x) = \sin(x + y - z) - x^2yz$

Find $\frac{\partial z}{\partial x}$

$\frac{-\frac{df}{dx}}{\frac{df}{dz}} = -\frac{\cos(x + y - z) - 2xyz}{-\cos(x + y - z) - x^2y}$

f

$x \leftarrow z$

$\frac{df}{dx} + \frac{df}{dz}\frac{dz}{dx} = 0$

$\frac{df}{dz}\frac{dz}{dx} = -\frac{df}{dx}$

$\frac{dz}{dx} = -\frac{\frac{df}{dx}}{\frac{df}{dz}}$

Know

$\frac{dr}{dt} = 1$ in/sec

$\frac{dh}{dt} = -2$ in/sec

Want

$\frac{dV}{dt} = ?$

when $r = 100$ in, $h = 40$ in

$$\frac{d}{dt}\left[V = \frac{1}{3}\pi r^2 h\right]$$

$$\frac{dV}{dt} = \frac{1}{3}\pi\left(2r\frac{dr}{dt}h + r^2\frac{dh}{dt}\right)$$

$$= \frac{1}{3}\pi\left(2\cdot 100\cdot 1\cdot 40 + -2\cdot 100^2\right)$$

$$= -4000 \text{ in}^3/\text{sec}$$

(3) $\sqrt{y+\cos^2 x} \approx 1 + \frac{1}{2}y$ @ $(0,0)$

$$\mathcal{L}(x,y) = f_x \cdot x + f_y \cdot y +$$

$$f_x = \frac{1}{2\sqrt{y+\cos^2 x}} \cdot -2\sin x\cos x \Big| f(x,y)$$